SOLUTION 13 FOR 6.013

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Solution 13.1

a) (Optional) \( f_{c,mn} = \frac{c}{2} \sqrt{(\frac{n}{a})^2 + \left(\frac{m}{b}\right)^2} \), the lowest four cutoff frequencies are:
   \( f_{c00} = 0[\text{Hz}] \), \( f_{c01} = f_{c10} = \frac{c}{2a} = 34\text{KHz} \), \( f_{c11} = \frac{\sqrt{2}c}{2a} = 48\text{KHz} \),
   \( f_{c20} = f_{c02} = 2F_{c01} = 68\text{KHz} \)

b) \( f_{c,mn} = \frac{c}{2} \sqrt{(\frac{n}{a})^2 + \left(\frac{m}{b}\right)^2 + \left(\frac{2n+1}{2d}\right)^2} \), the two lowest resonant frequencies are:
   \( f_{000} = \frac{c}{3d} = 4250\text{Hz} \), \( f_{001} = \frac{3c}{3d} = 12750\text{Hz} \)

c) Adults have lower cut-off frequencies than kids since the size of ear becomes bigger as people become older. Therefore, the hearing range of adults is between 0 Hz and 12 KHz. So, only one mode (cut off frequency = 0 Hz, lowest resonance frequency = 4250Hz) propagates in our ear.

d) (Optional) \( Z = c_s \rho A \), \( A \) is the cross area of tube. \( Z_0/Z_{air} = A_0/A_{air} = 0.5^2 / 5^2 = 1/100 \),
   \( |\Gamma|^2 = \left| \frac{Z_0 - Z_{air}}{Z_0 + Z_{air}} \right|^2 = \left| \frac{Z_0/Z_{air} - 1}{Z_0/Z_{air} + 1} \right|^2 = 0.96 \)

e) Only consider the wave reflected from the two ridges. If they are out of phase, this is null, in phase, peak.

   Null: \( d = \lambda/4, f = c_s/\lambda = c_s/4d = 8.5[\text{KHz}] \)

   Peak: \( d = \lambda/2, f = c_s/\lambda = c_s/2d = 17[\text{KHz}] \)

Some other frequencies also can produce nulls and peaks, but out of hearing range.

f) For the same frequency, waves from different direction will have different phase shift between two reflected waves from two ridges. This means the pattern of ear is not isotropic. Our cavemen ancestors can determine the direction of an unseen predator through the pattern.

g) (Optional) \( Q_E = \frac{w_n W_n}{P_n} = \frac{k}{2} K_n \frac{V}{A} \)
   \( k = \frac{1+\Gamma}{1-\Gamma} = 97.99 \)
   \( K_n = w/c_s = 2\pi \times 8500/340 = 157.1 \)
\[ V/A = a^2d/a^2 = d = 0.02 \]

So \( Q_E = 153.94 \)

Solution 13.2

a) \( \theta_c = \sin^{-1} \sqrt{\epsilon_1/\epsilon_2} = \sin^{-1} \sqrt{1.5/1.502} = 87^\circ \)

b) Get \( k_x \) in the cladding, \( k_x = \sqrt{k_c^2 + \alpha^2} \). According to the phase matching, \( k_g = k_x \), so \( \lambda_g = \frac{2\pi}{k_g} = \frac{2\pi}{k_x} = \frac{2\pi}{\sqrt{k_c^2 + \alpha^2}} = \frac{2\pi}{\sqrt{(6.41 \times 10^6)^2 + (10^6)^2}} = 0.968 \) microns, where \( k_c = 2\pi \sqrt{1.5}/\lambda = 6.41 \times 10^6 \)

Solution 13.3 (Optional)

a) State 4 has the smallest possibility to transit to other states, so state 4 is the most stable.

b) State 5 is the best state to pump.

c) 4 to 2 is the best transition. The photons in state 2 will drop to state 1 easily. Good for pumping again.

Solution 13.4 (Optional)

Please refer to equation (8.7.14) of text book, page 376: \( \frac{\Delta w}{w_0} = \frac{<\Delta w_m> - <\Delta w_e>}{W_t} \)

a) Short-circuited resonator has maximum current at ends. Indenting the ends will make \( <\Delta w_m> \) bigger than 0, the resonant frequency is upwards.

b) It is impossible to find the \( f_x \) to satisfy \( v = i \) at middle for an open-circuit resonator.

c) Open circuit has \( |voltage|_{max} \). Electric energy is max at \( |voltage|_{max} \), \( 2f_1 \) is the answer.